Class-X

Mathematics Standard (041)

Section–A

Quest To prove :- ORPQ is a square i.e. $\angle O = \angle Q = \angle P = \angle R = 90°$
and $OQ = QP = PR = OR$

Proof :- $\angle OQP = 90°$, $\angle ORP = 90°$] Tangent is perpendicular to the radius at the point of contact. —(1)

Now, In $\triangle ORP$ and $\triangle OQP$

$OP = OP$ (common)

$OR = OQ$ (Radii of same circle)

$PR = PQ$ (Tangents from an external point to a circle are equal)

Therefore, $\triangle ORP \cong \triangle OQP$ by SSS rule

$\therefore \angle OPR = \angle OPQ$ (By CPCT)

$\Rightarrow \angle OPR = \angle OPQ = 45°$

$\Rightarrow \angle OPR + \angle OPQ = 90°$

$\Rightarrow \angle QPR = 90°$ —(2)

In quad. OQPR

$\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360^\circ$ (Angle Sum Property)

$90 + 90 + 90 + \angle ROQ = 360^\circ$

$\angle ROQ = 360 - 270$

$\angle ROQ = 90^\circ$ — (3)

180
−90
270

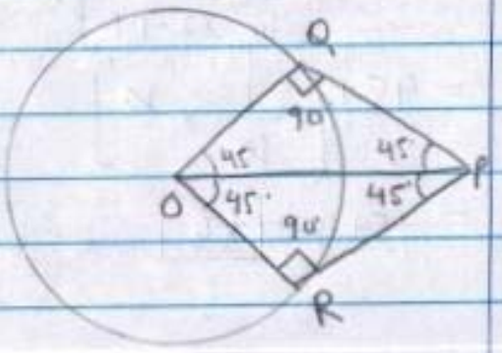
Also $OR = PR$ — (4)

(As $\angle ROP = \angle RPO = 45$) (Isosceles triangle property)

Also $QP = PR$ (Tangents from an external point to a circle are equal) — (5)

⇒ Using (4) and (5)

$OQ = PQ = PR = OR$ — (6)



Using (1), (2), (3), (6)

ORPQ is a square.

Hence proved.

Ques 2

Class		Frequency
15-25		6
25-35		11
35-45	f_0	22
45-55	f_1	23 → Modal class
55-65	f_2	14
65-75		5

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 45 + \left(\frac{23 - 22}{46 - 22 - 14}\right) \times 10$$

$$= 45 + \left(\frac{1}{10} \times 10\right)$$

$$= 45 + 1 = \boxed{46}$$

Ques 3

$a = 10$

$S_{14} = \frac{n}{2}(2a + (n-1)d)$

$1505 = \frac{14}{2}(2(10) + 13d)$

$1505 = 7(20 + 13d)$

$\frac{1505}{7} = 20 + 13d$

$215 = 20 + 13d$

$\frac{195}{13} = d$

$d = 15$

13 | 1505 | 115
-13
10
-7
35
-35
195
-195
0

13 | 195 | 15
-13
65
-65
0

Ques 4

9, 7, 5 ——— and 15, 12, 9 ———

$a_n = a + (n-1)d$

$= 9 + (n-1)-2$

$= 9 - 2n + 2$

$= 11 - 2n$

$a'_n = a' + (n-1)d'$

$= 15 + (n-1)-3$

$= 15 - 3n + 3$

$= 18 - 3n$

Since $a_n = a'_n$

$11-2n = 18-3n$

$11-18 = -3n+2n$

$-7 = -n$

$\boxed{n=7}$

Ques 5

(a) $x^2-2ax-(4b^2-a^2)=0$

$b^2-4ac = 4a^2-4\left[-(4b^2-a^2)\right]$ (1)

$= 4a^2-4\left[-4b^2+a^2\right]$

$= 4a^2+16b^2-4a^2$

$= 16b^2$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{2a\pm\sqrt{16b^2}}{2(1)} = \frac{2a\pm 4b}{2}$

$\Rightarrow x = \frac{2a+4b}{2} \quad x = \frac{2a-4b}{2}$

$x = a+2b \quad x = a-2b$

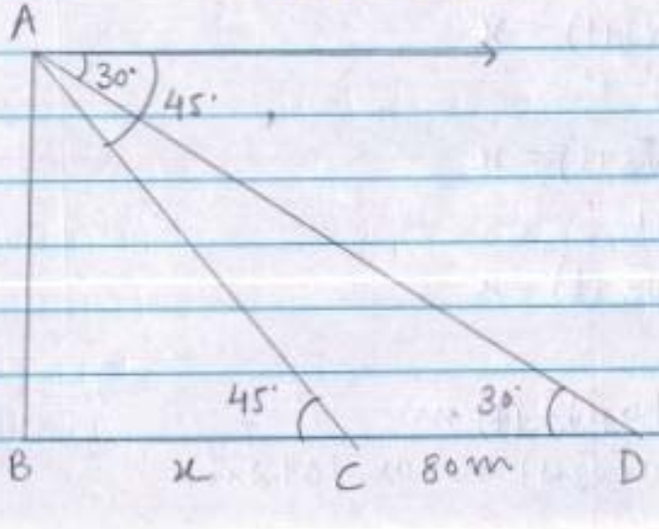
Ques 6

(b) Length of cuboid = 18cm
Breadth of cuboid = 6cm
Height of cuboid = 6cm

TSA of the cuboid = $2(lb + bh + hl)$
$= 2(18 \times 6 + 6 \times 6 + 6 \times 18)$
$= 2(108 + 36 + 108)$
$= 2(252)$
$= 504\ cm^2$

Section-B

Ques 7



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8

In ΔABC

$\tan 45 = \frac{P}{B} = \frac{AB}{x}$

$1 = \frac{AB}{x}$

$\boxed{AB = x}$

In ΔABD

$\tan 30 = \frac{P}{B} = \frac{AB}{80+x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{80+x}$

$\frac{80+x}{\sqrt{3}} = AB$

$\frac{80+x}{\sqrt{3}} = x$

$80 + x = \sqrt{3}x$

$80 = \sqrt{3}x - x$

$80 = x(\sqrt{3}-1)$

$\frac{80}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = x$

$\frac{80(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2} = x$

$\frac{80(\sqrt{3}+1)}{3-1} = x$

$\frac{80(\sqrt{3}+1)}{2} = x$

$x = 40(\sqrt{3}+1)$ m

$\Rightarrow AB = 40(\sqrt{3}+1)$ m or 109.2 m

(Rough work: 1.73 + 1 = 2.73; 2.73 × 4 = 10.92; 2.73 × 40 = 109.20)

Ques 8

Class	Frequency	C.F.
1400-1550	6	6
1550-1700	13	19
1700-1850	25	44 → Median class
1850-2000	10	54 = n

$$\text{Median} = l + \left(\frac{\frac{n}{2} - CF}{f}\right) \times h$$

$$= 1700 + \left(\frac{27 - 19}{25}\right) \times 150$$

$$= 1700 + \left(\frac{8}{25}\right) \times 150^{6}$$

$$= 1700 + 48$$

$$= 1748$$

13
6
19
25
44

$\frac{54}{2} = 27$

1850
1700
150

1271
-19
8

19
25
44

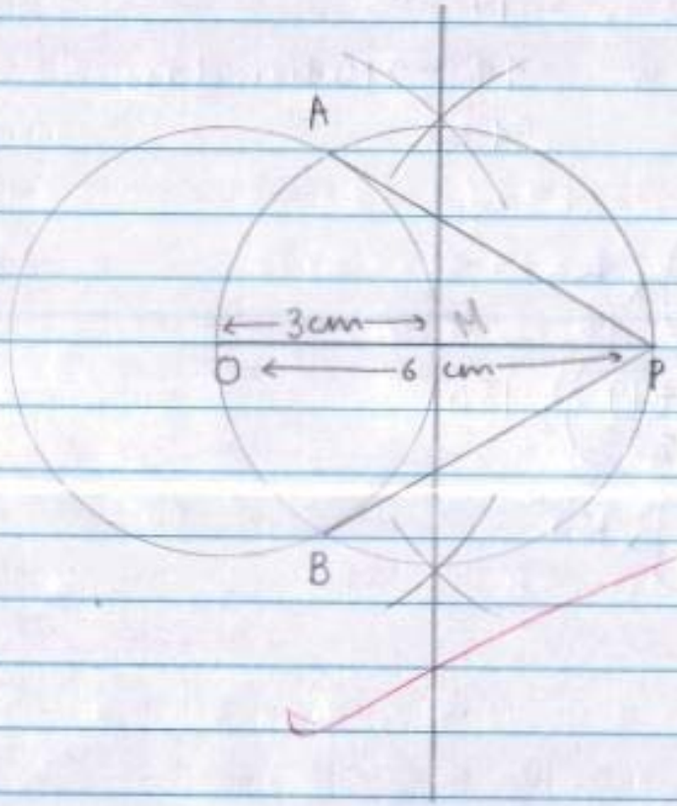
1850
-1700
150

$\frac{54}{2} = 27$

1271
-19
8

Ques 9

(b)



Steps of Construction →

1. Draw a circle with O as centre and radius 3cm.
2. Take a point P outside the circle at a distance of 6cm from its centre O.
3. Join OP.
4. Construct the perpendicular bisector of OP.
5. Name the mid-point of OP as M.
6. Draw a circle with M as centre and radius OM.
7. The second circle intersects the first circle at two points. Name them A and B.
8. Connect PA and PB.
9. PA and PB are the required tangents.

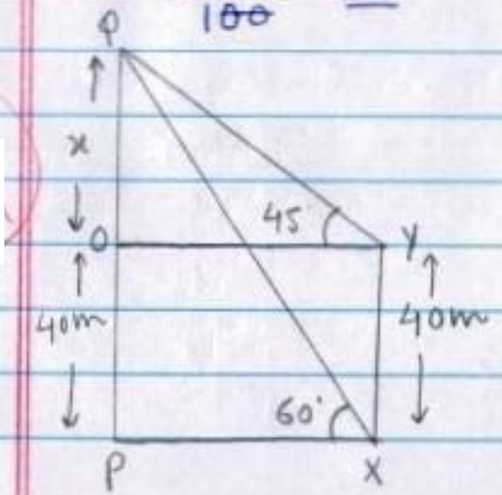
Ques 10

Class	Frequency (f_i)	x_i	$f_i x_i$
0–10	12	5	60
10–20	18	15	270
20–30	27	25	675
30–40	20	35	700
40–50	17	45	765
50–60	6	55	330
	100		2800

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\bar{x} = \frac{2800}{100} = 28$$

Ques H



In ΔQOY

$$\tan 45 = \frac{P}{B} = \frac{QO}{OY} = \frac{x}{OY}$$

$$1 = \frac{x}{OY}$$

$$\boxed{OY = x} \Rightarrow \boxed{Px = x}$$

In ΔQPX

$$\tan 60 = \frac{P}{B} = \frac{QP}{PX} = \frac{40+x}{x}$$

$$\sqrt{3} = \frac{40+x}{x}$$

$$\sqrt{3}x = 40+x$$

$$\sqrt{3}x - x = 40$$

$$x(\sqrt{3}-1) = 40$$

$$x = \frac{40}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{40(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2} = \frac{40(\sqrt{3}+1)}{3-1}$$

$$x = \frac{40(\sqrt{3}+1)}{2}$$

$$x = 20(\sqrt{3}+1)\ m$$

$$x = 54.6m$$

$$\Rightarrow \boxed{PX = 54.6m}$$

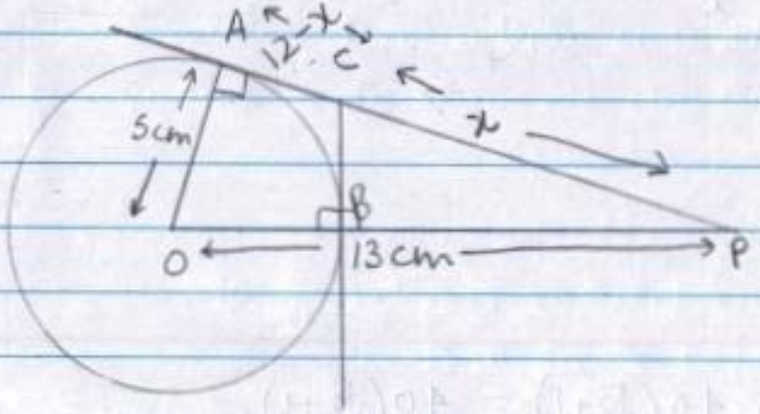
$$PQ = 40 + x$$

$$= 40 + 54.6$$

$$= \boxed{94.6m}$$

Ques 12

(b)



$\angle OAP = 90°$ (Tangent is perpendicular to radius at the point of contact)

Using pythagoras theorem in $\triangle OAP$

$H^2 = B^2 + P^2$

$OP^2 = AP^2 + OA^2$ $\Rightarrow AP^2 = 144$

$13^2 = AP^2 + 5^2$ $\boxed{AP = \sqrt{144} = 12\,cm}$

$169 = AP^2 + 25$

$169 - 25 = AP^2$

Let PC be x and AC be 12-x

AC = BC = 12-x (Tangents from point C to the circle are equal)

Also OP = 13 cm

OB = 5cm

BP = OP - OB

BP = 8cm

$\angle OBC = 90^\circ$ (Tangent is perpendicular to the radius at the POC)

$\Rightarrow \angle CBP = 90^\circ$ (Linear Pair)

Using pythagoras theorem in Δ CBP

$H^2 = B^2 + P^2$

$CP^2 = BP^2 + BC^2$

$x^2 = 8^2 + (12-x)^2$

$x^2 = 64 + 144 + x^2 - 24x$

$24x = 64 + 144$

$24x = 208$

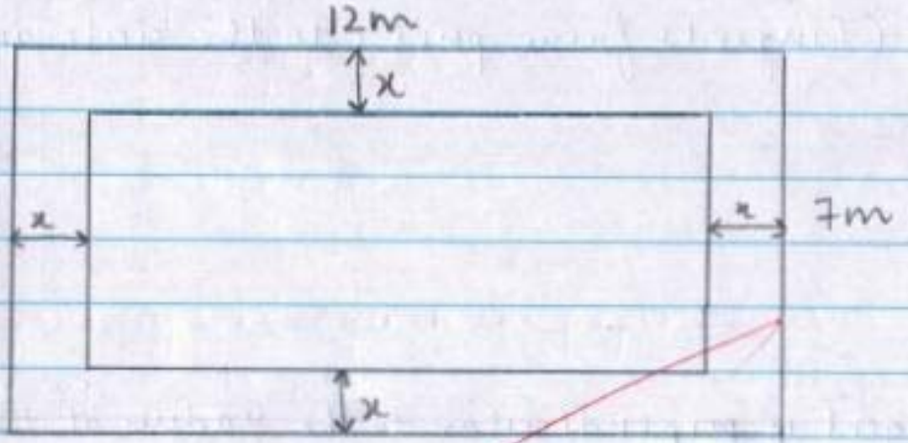
$x = \frac{208}{24} \frac{104}{12} \frac{52}{6} \frac{26}{3}$

$BC = 12 - x$

$BC = 12 - \frac{26}{3}$

$BC = \frac{36 - 26}{3} = \frac{10}{3} = 3.34\ m$

Ques 13



~~Length of pool = (12-2x) m~~ Dimensions of pool —
~~Breadth of pool = (7-2x) m~~ (12-2x) and (7-2x)

(a) Area = length x Breadth

$36 = (12-2x)(7-2x)$

$36 = (84 - 24x - 14x + 4x^2)$

$36 = 84 - 38x + 4x^2$

$36 - 84 = -38x + 4x^2$

$-48 = -38x + 4x^2$

$4x^2 - 38x + 48 = 0 \Rightarrow 2x^2 - 19x + 24 = 0$

(b) $2x^2 - 19x + 24 = 0$

$2x^2 - 16x - 3x + 24 = 0$

$2x(x-8) - 3(x-8) = 0$

$x = \frac{3}{2}$ $x = 8$

$x = 8$ will be neglected because if $x = 8$ the width of sidewalk will become more than 16 but they are already given 7m and 12m, the dimensions

$\therefore \boxed{x = 1.5m}$

48
1×48
2×24
3×16

24
×24
96
48
576
49
625

Ques 14

(a) CSA of one cap = Square cm paper required for one cap

$2\pi r = 44$

$2 \times \frac{22}{7} \times r = 44$

$r = \frac{44 \times 7}{22 \times 2} = 7cm$

$l^2 = h^2 + r^2$

$l^2 = 24^2 + 7^2$

$l^2 = 576 + 49$

$l^2 = 625$

$l = 25cm$

CSA of one cap $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 550\ cm^2$

CSA of four caps ie. square cm paper required for four caps $= 4 \times 550$

$= 2200\ cm^2$

(b) Volume of cake $= \pi r^2 h$

$= \frac{22}{7} \times 12 \times 12 \times 14.2$

$d = 24\ cm$

$r = \frac{d}{2} = 12\ cm$

$= 6336\ cm^3$

$650\ cm^3 = 100g$ or $0.1kg$

$6336\ cm^3 = \frac{0.1kg \times 6336}{650} = \frac{6336}{6500} = \frac{95.9}{100} = 0.95kg$

So they should order a 1 kg cake.